# SIMULTANEOUS EFFECT OF A MOVING LOAD AND A GROUND MOTION ON A SIMPLE BEAM: NUMERICAL APPROACH

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## ABSTRACT

The paper deals with a simultaneous effect of a row of regularly distributed moving loads and a vertical seismic ground excitation on a single-span bridge. The simple mathematical model is used to allow direct comparison of analytical and numerical results. Advantages and drawbacks of both methods are discussed. Various levels of simplification of description the loads are supposed. The discussed methodology is illustrated on an example of an interaction of the real concrete bridge, the high-speed train TALGO AV2 and an artificial earthquake data.

***KEYWORDS: MOVING FORCES, SUPPORT MOTION, FINITE DIFFERENCES***

## Introduction

The recent development and expansion of the high-speed railway services puts additional demands on the railway tracks, their design, construction and overall safety. This holds particularly true for bridges. The problem of the dynamic action of moving loads on structures as well as the specific effect of the high-speed rail traffic on bridges is reported regularly in the scientific literature and design codes, e.g. [Frýba (1996)] or [Frýba and Fischer (2003)], however, the combined effect of train and earthquake attracted attention only recently, see [Yau and Frýba (2005)]. In their first and also in the subsequent works authors [Yau and Frýba (2007)] or [Frýba and Yau (2009)] and finally [Frýba *et al.* (2014)] limit themselves on the case of the vertical vibration of a beam subjected to a row of moving forces. The supports of the beam are simultaneously supposed to provide vertical movements due to an earthquake. This model is unrealistic under certain circumstances, as the authors admit, from the point of view of both earthquake and bridge model. The effect of an earthquake is usually supposed to be dominant in the horizontal direction; only several harmonic components of the earthquake record are taken into account and the bridge is modelled as a simply supported Euler beam. However, simplicity of the model allows to formulate the analytic solution, which is able to provide a qualitative insight into the problem. Such an analytic solution can be favourably used for validation of a numerical procedure, which can be subsequently easily extended for a more general model.

It has to be mentioned that the analytical formulae, which were derived for action of moving load on structures in, e.g., [Frýba (1996)] and which were extended for the combined problem in the above cited works by Frýba and Yau, bring their own difficulties for numerical enumeration: they involve partial sums of infinite trigonometric series, which can introduce spurious oscillations, or hidden pairs of terms, which cancel themselves under certain

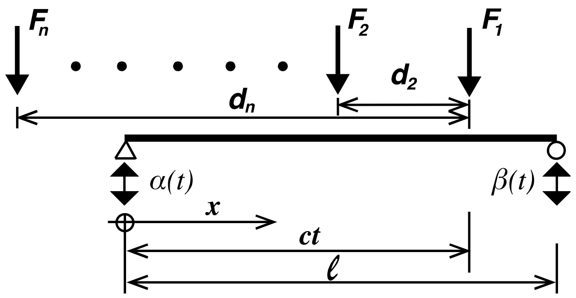

Figure 1: Theoretical model of a beam, moving forces and support movements.

conditions and thus they are a potential source of numerical instability. Moreover, simplifying assumptions like lack of damping, rough model of an earthquake process or a limited number of eigenforms taken into account lower credibility of the formulae. Such obstacles introduce additional arguments supporting the numerical approach.
There is a wide range of numerical techniques available to solve the fourth order parabolic partial differential equations (PDEs). The traditional discretization methods comprise explicit and implicit finite difference schemas or several variants of finite element methods. Another popular option is the method of lines, which reformulates the PDE to the form convenient for application of a standard ordinary differential equation (ODE) solver. In this paper, the implicit difference schema is used, which was introduced for this particular problem in [Fischer *et al.* (2014)]

**Mathematical Model**

The dynamic action of the combined load on a simply supported beam is described by the fourth order PDE. The beam of span $\ell$ is subjected to a row of moving forces $F_n$, $n=1,2,3,\ldots,N$ at the distances $d_n$, see the Figure 1. The forces are moving from the left to the right with a constant speed $c$. The supports of the beam perform the vertical movements $\alpha(t)$ (left support) and $\beta(t)$ (right support), respectively. The equation together with initial and boundary conditions reads:

$$EI\, v^{iv}(x,t) + \mu\, \ddot{v}(x,t) + 2\mu\gamma\, \dot{v}(x,t) = \sum_{i=1}^{n} F_i\, \varepsilon_i(t)\delta(x-d_i) \quad (1)$$

$$v(0,t) = \alpha(t), v(\ell,t) = \beta(t), \quad v''(0,t) = 0, v''(\ell,t) = 0 \quad (2)$$

$$v(x,0) = \dot{v}(x,0) = 0 \quad (3)$$

where it has been denoted:
$v(x,t)$ – vertical displacement of the beam at $x$ and time $t$, respectively,
$EI$ – flexural rigidity of the beam (constant),
$\mu$ – mass per unit length of the beam (constant),
$\gamma$ – circular frequency of the beam damping,
$\varepsilon_i(t) = h(t-t_i) - h(t-T_i)$, where $h(t)$ is the Heaviside unit step function,
$\delta(x)$ – Dirac function,
$t_i = d_i/c, T_i = (\ell + d_i)/c$ – time when the $i$-th force enters or leaves the beam
$d_i$ – distance between the first and $i$-th force $d_1 = 0$,
$\bullet', \dot{\bullet}$ – differentiation with respect to space or time, respectively.

The boundary conditions (2) characterize the “simply supported beam” with prescribed movement of its both ends. The both soil displacement functions are usually assumed to be equal $\alpha(t) = \beta(t)$ or shifted $\alpha(t) = \beta(t \pm \Delta t)$ on both ends, however, the general choice $\alpha(t) \neq \beta(t)$ is supposed here. The initial condition (3) assumes that the structure is initially at rest.

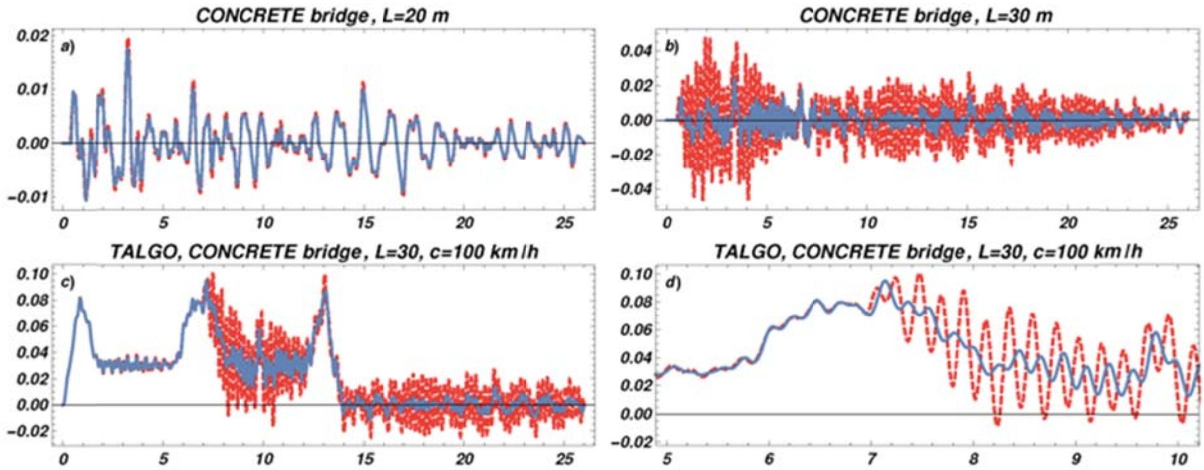


Figure 2: Response of the mid-span point of the concrete bridges due to (a-b) earthquake only, (c-d) earthquake and train action. Dark solid lines: numerical solution, red dashed lines: analytical expression. In plots (c-d) the earthquake shock starts at $t$=7s, when the middle car of the TALGO AV2 train ($c$=100km/h) leaves the bridge. Plot (d) is detail of plot (c) for t=5-10s.
Bridge parameters ( $f_1$– first eigenfrequency, $\vartheta$ –damping, $G$ – total weight ):
(a) $\ell$=20m, $f_1$=7.04Hz, $\vartheta$=0.18, $G$=800kN; (b-d) $\ell$=30m, $f_1$=4.51Hz, $\vartheta$=0.18, $G$=800kN

**Closed Form Solution**

The closed form solution to the problem of the beam vibration (1-3) used in this work follows the standard resolution into the *quasi-static component* comprising the variable boundary conditions and the zero right hand side and the *dynamic component* which includes the moving load. The time-dependent boundary conditions $\alpha(t),\ \beta(t)$ of the quasi-static part are assumed to be represented by a sum of several selected (dominant) terms of a finite Fourier approximation of the earthquake process, possibly modulated by a function of "slow time". This quasi-static component is then solved using the Fourier method, whereas the dynamic component is resolved using the eigenform expansion. The procedure is described in detail in [Frýba *et al.* (2014)].

**Numerical Solution**

The numerical solution to the system (1-3) exploits the finite difference scheme of the 4$^{th}$ order for discretization in the space variable and the 2$^{nd}$ order implicit Euler scheme for interpolation in time. This approach is simple enough to allow to easily implement possible alterations and, due to continuous nature of the problem, it offers a reasonable accuracy. The target formula is derived in full in [Fischer *et al.* (2014)] and due to space limitation will not be presented here.

The implicit Euler scheme is known to be unconditionally stable, however, it introduces an artificial numerical dispersion (damping), see, e.g., [Hoffman (2001)]. The level of numerical dispersion depends on value $b = \varDelta\sqrt{(EI/\mu)}/h^2$ , where $\varDelta, h$ are (uniform) discretization steps in time and space, respectively.

The discretization parameters $\varDelta, h$ should also be chosen in such a way as to allow the consistent description of the moving load. The value of $h$ should correspond to axle distances of the supposed train and the time step $\varDelta$ has to be dependent on the train velocity. Attention should be paid to consistent distribution of the axle load $F_i$ between two adjacent space nodes.

**Results**

Although the numerically computed response generally corresponds well to the one obtained using the explicit formula, for particular values of the parameters can the results differ significantly. Figures 2(a-b) show sample response of two concrete bridges of lengths $\ell$=20m and 30m due to an earthquake action. Whereas for the shorter bridge the analytical and numerical responses do not differ, the analytical solution for the longer bridge (b) exhibits spurious resonance oscillations which do not occur in the numerical solution. Plots (c-d) show the vertical response of the mid-span point of the bridge $\ell$=30m due to the combined loading of an earthquake and the train Talgo AV2. The artificial accelerogram based on 65 dominant frequency components of the famous El Centro 1940 record is used for the both analytical and numerical approaches for the left end of the beam $\alpha(t)$, the other end is supposed to be in rest $\beta(t) = 0$. In the presented example the earthquake shock starts when the middle car of the train leaves the bridge. At this moment is the response due to passing train maximal, as the four middle axles forces of the Talgo AV2 train represent a pair of engines. Spurious oscillations in the analytical solution are dominant even in this case of combined loading.

## Conclusions

The high-speed trains substantially affect the dynamic behaviour of railway bridges, which could be brought even to the resonant vibration. It is caused by a long sequence of axle forces or their groups distributed in almost regular distances. Earthquake, as a broadband process, can induce similar effect on the bridge. Combination of the earthquake shock and the high-speed train load brings new demands on the properties of the structure. It comes to light that the oversimplification of the complex problem can cause false predictions of the analytical models. On the other hand, numerical dispersion is an inherent property of recursive numerical procedures, which can cause the underestimation of the response. The both facts attract attention of the research to development of new approaches, either based on stochastic description of the earthquake process, or some kind of numerical stabilization of the numerical integration procedure.

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