EXPERIMENTAL AND THEORETICAL STUDIES OF INDIRECT BRIDGE FREQUENCY MEASUREMENT USING A PASSING VEHICLE

Shota Urushadze[1], J.D. Yau[2], and Ladislav Fryba[3]

[1]*Head of Department, Institute of Theoretical and Applied Mechanics, v.v.i., ASCR, Prague, Czech Republic.*
urushadze@itam.cas.cz
[2] *Professor, Department of Civil Engineering, Tamkang University, Tamsui, New Taipei City, Taiwan.*
[3] *Professor, Institute of Theoretical and Applied Mechanics, v.v.i., ASCR, Prague, Czech Republic.*

ABSTRACT

In this paper, an indirect bridge frequency monitoring method will be presented through theoretical formulation and experimental verification. The idea of indirect method is based on the coupling nature of a vehicle moving on a bridge proposed by Yang et al. (2004), in which the bridge response can be recorded by the passing vehicle. Thus, one can extract the bridge frequencies from the response of the moving instrumented vehicle. With this concept, this study adopted a simplified vehicle-bridge interaction model to present a semi-analytical solution of dynamic response for a vehicle traveling on a simply supported beam, from which the vibration data of bridge response was recorded in the instrumented vehicle. Then an experimental setup for measuring the bridge frequency was introduced for verification of the indirect method. From the present experimental results, the indirect bridge inspection method is applicable to monitor the dynamic characteristics of a bridge.

KEYWORDS: BRIDGE HEALTH MONITORING, INDIRECT METHOD, MOVING LOAD, VBI

Introduction

Conventional bridge inspection needs to install a lot of sensors on a bridge directly. This approach is called direct bridge monitoring method in this study. To simplify the bridge monitoring procedure in practice, Yang et al. [2004] proposed a vehicle-bridge interaction (VBI) model to extract beam frequencies from the response of a passing sprung mass unit based on a quasi-closed form solution of the VBI system. This approach is referred to *indirect monitoring method*. For a VBI system, the passing vehicle can be regarded as an active actuator to excite the bridge and also as a response receiver to capture the vibration data from the vibrating bridge. Then the vibration data recorded by the passing vehicle can be used to *extract* the bridge frequencies. In this study, a simplified vehicle-bridge interaction system is represented by a moving sprung mass unit over a simple beam so that one can derive a semi-analytical solution of dynamic response for the sprung mass traveling on the beam, from which the vibration data of bridge response was recorded in the instrumented vehicle. Then an experimental setup for measuring the bridge frequency was introduced for verification of the indirect method. From the experimental verifications, the indirect method shows an excellent efficiency and mobility in assessment techniques appropriate for bridge health monitoring.

Semi-analytical solution of a VBI system

To derive the dynamic response of a sprung mass unit traveling over a simple beam in analytical form, we consider the model shown in Fig. 1 with following assumptions [Fryba 1999]:

(1) The simple beam is considered as a linearly elastic Bernoulli-Euler beam with smooth surface;

(2) An undamped sprung mass unit moving at constant speed v is adopted in generating the closed-form dynamic response of the simple beam;

(3) The inertial force of the sprung mass on the response of the beam is neglected in determining the response of the beam, namely, only the static weight of the sprung mass is considered in this regard.

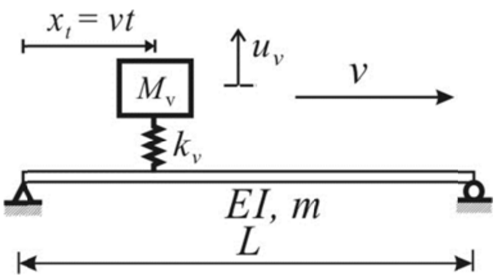

Fig. 1 Schematic diagram of an un-damped VBI system

In Fig. 1, the following parameters are adopted for the beam: m = mass per unit length, L = span length, EI = flexural rigidity, and the following for the sprung mass unit: v = moving speed, M_v, = lumped mass, and k_v = spring stiffness. We can write the equations of motion for the beam and the sprung mass moving over the beam as [Yang et al. 2004]:

$$m\ddot{u} + EIu'''' = -\left(p_0 - M_v \ddot{u}_v\right)\delta\left(x - vt\right) \quad 0 \le t \le L/v \tag{1}$$

$$M_v \ddot{u}_v + k_v u_v = k_v u(x_t, t) \tag{2}$$

where $(\bullet)' = \partial(\bullet)/\partial x$, $(\dot{\bullet}) = \partial(\bullet)/\partial t$, $u(x,t)$ = vertical deflection of the beam, u_v = vertical displacement of the sprung mass, p_0 = $M_v g$ = weight of the sprung mass, g = gravity acceleration, L = span length, $\delta\left(\bullet\right)$ = Dirac's delta function, $H(t)$ = unit step function, and x_t = vt = the position of the moving sprung mass on the beam. For a simply supported beam, the following boundary conditions are adopted:

$$u(0,t) = u(L,t) = 0, \quad EIu''(0,t) = EIu''(L,t) = 0 \tag{3}$$

for hinge or roller supports.

As indicated in Eq. (2), the moving sprung mass will be excited by the motion $u(x,t)$ of the beam directly in contact. From the viewpoint of practical bridges, the mass of a bridge is much larger than that of a running vehicle. It was demonstrated that neglecting the inertial force ($M_v \ddot{u}_v$) of the sprung mass will not affect the response of the beam. With this assumption, Eq. (1) can be simplified as follows:

$$m\ddot{u} + EIu'''' \approx -p_0 \delta\left(x - vt\right) \tag{4}$$

with the boundary conditions of simply supported ends, , as shown in Eq. (3). Let us consider the first vibration mode of the simple beam, the deflection $u(x,t)$ of the beam subjected to a moving static force can be approximated as [Yang et al. 2004]

$$u(x,t) \approx \sum_{n=1} \Delta_s \left[\sin\frac{\pi vt}{L} - S_1 \sin \omega_b t\right] \times \sin\frac{\pi x}{L} \tag{5}$$

in which the speed parameter S_1 is defined as

$$\Delta_{s1} = \frac{-2p_0L^3}{EI\pi^4\left(1-S_1^2\right)} \square \frac{\Delta_{st}}{\left(1-S_1^2\right)}, \quad \Delta_{st} = \frac{-p_0L^3}{48EI}, \quad S_1 = \frac{\pi v}{\omega_b L}, \omega_b = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{m}} \tag{6}$$

Substituting the beam deflection shown in Eq. (5) into Eq. (2) yields

$$M_v\ddot{u}_v + k_v u_v = k_v u(x_t,t) = k_v\Delta_{s1}\left[\left(\sin\frac{\pi vt}{L}\right)^2 - S_1\sin\frac{\pi vt}{L}\sin\omega_b t\right] \tag{7}$$

The excitational force on the right-hand side of Eq. (7) consists of two parts: The first term with $\sin(n\pi vt/L)$ is the *driving force*, and the second term with $\sin(\omega_b t)$ represents the *free vibration* of the beam. By solving the differential equation of Eq. (7) with zero initial conditions, one can obtain the displacement response of the sprung mass as

$$\begin{aligned} u_v(t) \square\ & \frac{\Delta_{s1}}{2}\frac{S_v^2}{1-S_v^2}\left[\cos\left(\frac{2\pi}{L}vt\right)+\cos\left(\omega_v t\right)\right] \\ & + \frac{\Delta_{s1}S_1}{2}\left[C_{b1}\cos\left(\omega_b t - \frac{\pi vt}{L}\right) + C_{b2}\cos\left(\omega_b t + \frac{\pi vt}{L}\right) + C_v\cos(\omega_v t)\right], \end{aligned} \tag{8}$$

where $S_v = \dfrac{2\pi v}{\omega_v L}$ and

$$C_{b1} = \frac{-1}{1-\left(\dfrac{\omega_b}{\omega_v}-\dfrac{S_v}{2}\right)^2}, C_{b2} = \frac{1}{1-\left(\dfrac{\omega_b}{\omega_v}+\dfrac{S_v}{2}\right)^2} \quad C_v = \frac{1}{\left[\omega_v^2-\left(\omega_b-\dfrac{\pi v}{L}\right)^2\right]} - \frac{1}{\left[\omega_v^2-\left(\omega_b+\dfrac{\pi v}{L}\right)^2\right]} \tag{9}$$

As shown in Eq. (8) and (9), the frequency content of the vehicle response is composed by three components: vehicle frequency (ω_v), bridge frequency (ω_b) and driving frequency ($2\pi v/L$). Obviously, the dynamic response of a passing vehicle over a bridge is easier to measure than to install a lot of sensors on a bridge for vibration measurement. Therefore, the indirect bridge monitoring method becomes very attractive for bridge engineers to perform bridge health monitoring. In the following section, a VBI experimental setup will be introduced and used to verify the feasibility of the indirect method.

Experimental setup

A simple laboratory model of the investigated problem was constructed. Simulation of the passing vehicle was carried out on a plexiglass beam with length 2m, see figure 2. The response to the moving mass was measured during the passing over the beam and for the verification of dynamic parameters of the beam was also measured at several places. The experimental test set-up is shown in Figs. 3. The set-up allows change the speed of crossing and stiffness parameters of the vehicle.

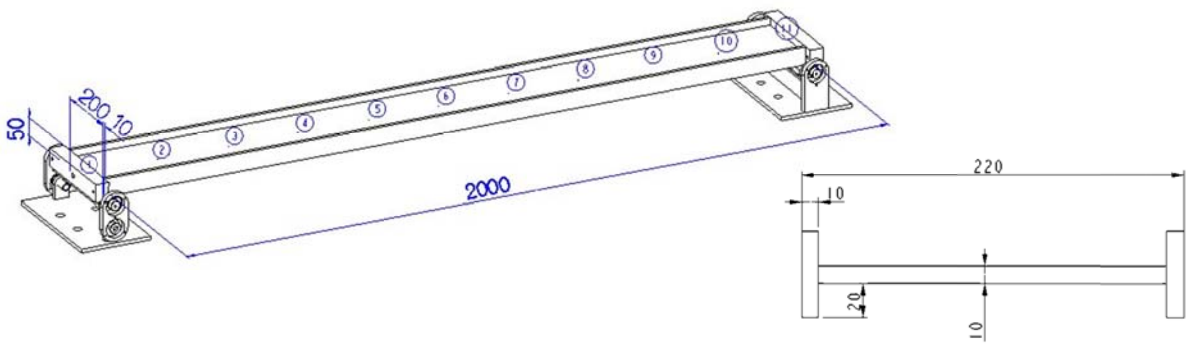

Fig. 2. Laboratory model and cross-section of the beam

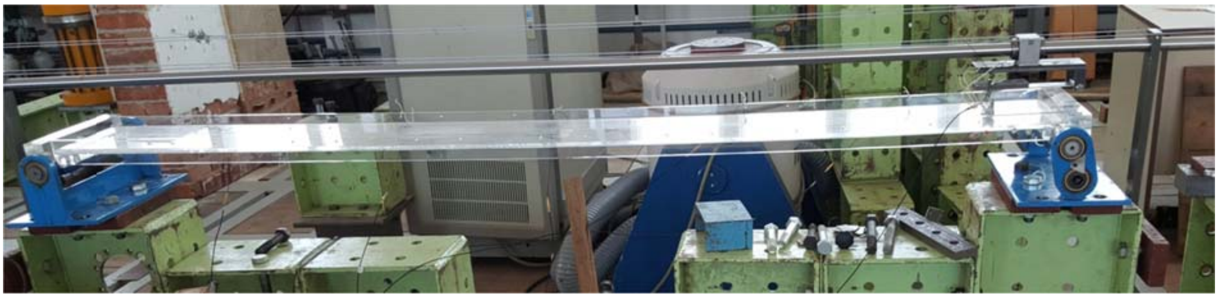

Fig. 3. The test set up

Discussion of test results

The first natural frequencies f of beam is 6.348 Hz. Some interesting results were already obtained by the signal processing on the passing vehicle, which showed a frequency corresponding of first natural frequency of the beam. The results are shown for different spring stiffness in Fig. 4.

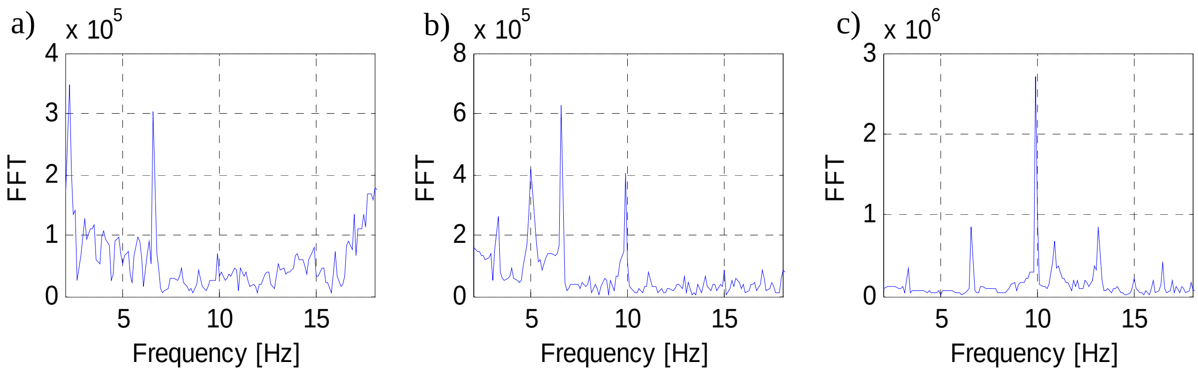

Fig.4. Frequency analysis for various springs
a) soft spring, b) middle spring and c) hard spring

Conclusions

The theoretical study and experiments carried out by the authors showed the possibilities using the presented indirect method to identify the dynamic characteristics of the bridge, e.g. the natural frequency. Moreover, a harder spring in the vehicle was used for experiments and it presented better results than the softer ones.

Acknowledgements

The research reported herein was sponsored in part by grants from the Ministry of Science and Technology in Taiwan through the serial No. 103-2221-E-032-020-MY3 and the Czech Science Foundation, GACR project 13-34405J. Such financial supports are gratefully acknowledged.

References

Fryba, L. (1999). Vibration of Solids and Structures under Moving Loads. Thomas Telford, 3-rd ed., London.

Yang, Y.B., Lin, C.W., and Yau, J.D. (2004), “Extracting bridge frequencies from the dynamic response of a passing vehicle, Journal of Sound and Vibration, 272(3-5), 471–493.